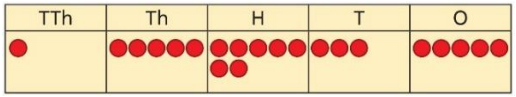
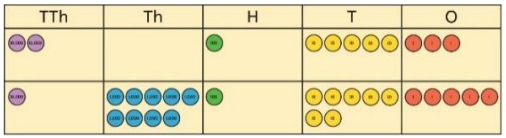
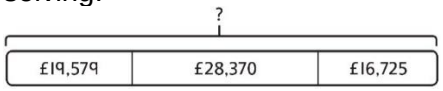
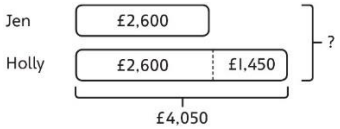
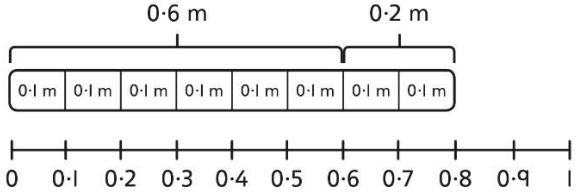
KEY STAGE 2
Key language: decimal, column methods, exchange, partition, mental method, ten thousand, hundred thousand, million, factor, multiple, prime number, square number, cube number, product, divisor, factor, quotient,

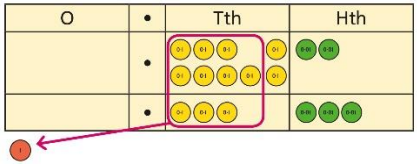
Year 5			
	Concrete	**Pictorial**	**Abstract**
Year 5 Addition	• **Teaching point 1**: Mathematical relationships encountered at primary level are either additive or multiplicative; both of these can be observed within the structure of part–part–whole relationships. • **Teaching point 2**: Problems in many different contexts can be solved by adding together the parts to find the whole. Different strategies can be used to calculate the whole, but the structure of the problem remains the same. • **Teaching point 3**: If the value of the whole is known, along with the values of all but one of the parts, the value of the missing part can be calculated. Different strategies can be used to calculate the missing part, but the structure of the problem remains the same. • **Teaching point 4**: Problems in many different contexts have the 'missing-part' structure. • **teaching point 5**: If one addend is increased and the other is decreased by the same amount, the sum stays the same. (same sum) • **Teaching point 6**: If one addend is increased (or decreased) and the other is kept the same, the sum increases (or decreases) by the same amount. • **Teaching point 7**: The value of the expressions on each side of an equals symbol must be the same; addition and subtraction are inverse operations. We can use this knowledge to balance equations and solve problems.		
Column addition with whole numbers	Use place value equipment to represent additions. *Add a row of counters onto the place value grid to show 15,735 + 4,012.*	Represent additions, using place value equipment on a place value grid alongside written methods.	Use column addition, including exchanges.

		 *I need to exchange 10 tens for a 100.* TTh Th H T O 2 0 1 5 3 + 1 9 1 7 5 3 9 3 2 8	TTh Th H T O 1 9 1 7 5 + 1 8 4 1 7 3 7 5 9 2
Representing additions		Bar models represent addition of two or more numbers in the context of problem solving.   Th H T O 2 6 0 0 + 1 4 5 0 4 0 5 0 Th H T O 2 6 0 0 + 4 0 5 0 6 6 5 0	Use approximation to check whether answers are reasonable. TTh Th H T O 2 3 4 0 5 + 7 8 9 2 2 0 2 9 7 TTh Th H T O 2 3 4 0 5 + 7 8 9 2 3 1 2 9 7 *I will use 23,000 + 8,000 to check.*
Adding tenths	Link measure with addition of decimals. *Two lengths of fencing are 0·6 m and 0·2 m.* *How long are they when added together?* 	Use a bar model with a number line to add tenths.  *0·6 + 0·2 = 0·8* *6 tenths + 2 tenths = 8 tenths*	Understand the link with adding fractions. $\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$ *6 tenths + 2 tenths = 8 tenths* *0·6 + 0·2 = 0·8*

<table>
<tr>
<td>Adding decimals using column addition</td>
<td>Use place value equipment to represent additions.

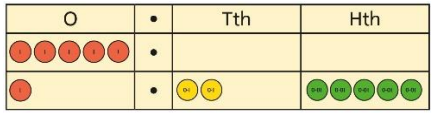
Show 0·23 + 0·45 using place value counters.</td>
<td>Use place value equipment on a place value grid to represent additions.

Represent exchange where necessary.

O · Tth Hth
0 · 9 2
+ 0 · 3 3
1 · 2 5
1

Include examples where the numbers of decimal places are different.

O · Tth Hth
5 · 0 0
+ 1 · 2 5
6 · 2 5</td>
<td>Add using a column method, ensuring that children understand the link with place value.

O · Tth Hth
0 · 2 3
+ 0 · 4 5
0 · 6 8

Include exchange where required, alongside an understanding of place value.

O · Tth Hth
0 · 9 2
+ 0 · 3 3
1 · 2 5
1

Include additions where the numbers of decimal places are different.

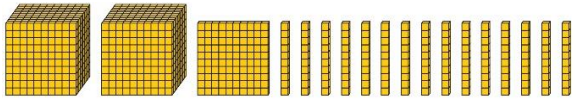
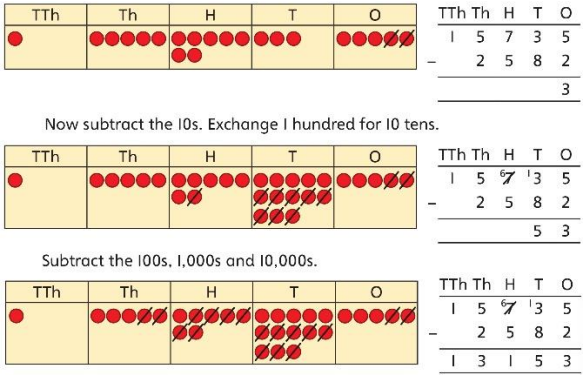
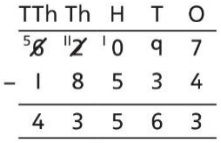
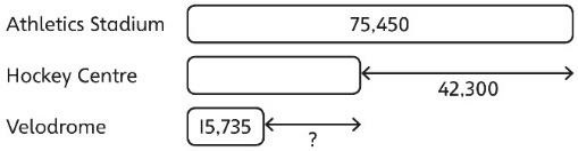
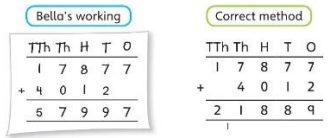
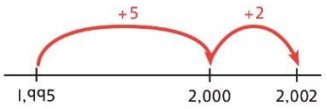
3.4 + 0.65 = ?

O · Tth Hth
3 · 4 0
+ 0 · 6 5
·</td>
</tr>
<tr>
<td>Year 5 Subtraction</td>
<td colspan="3">

Teaching point 1: If the minuend and subtrahend are changed by the same amount, the difference stays the same. (same difference)
Teaching point 2: If the minuend is increased (or decreased) and the subtrahend is kept the same, the difference increases (or decreases) by the same amount.

Teaching point 3: If the minuend is kept the same and the subtrahend is increased (or decreased), the difference decreases (or increases) by the same amount.
</td>
</tr>
<tr>
<td>Column subtraction with whole numbers</td>
<td>Use place value equipment to understand where exchanges are required.

2,250 – 1,070</td>
<td>Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required.</td>
<td>Use column subtraction methods with exchange where required.</td>
</tr>
</table>

		*15,735 − 2,582 = 13,153*  	 *62,097 − 18,534 = 43,563*
Checking strategies and representing subtractions		Bar models represent subtractions in problem contexts, including ‘find the difference’.  	Children can explain the mistake made when the columns have not been ordered correctly.  Use approximation to check calculations. *I calculated 18,000 + 4,000 mentally to check my subtraction.*
Choosing efficient methods			To subtract two large numbers that are close, children find the difference by counting on. *2,002 − 1,995 = ?*  Use addition to check subtractions. *I calculated 7,546 − 2,355 = 5,191.* *I will check using the inverse.*

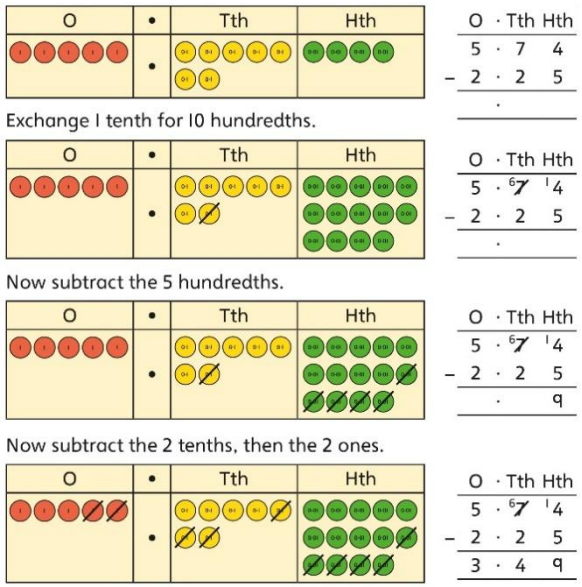
<table>
<tr>
<td>Subtracting decimals</td>
<td>Explore complements to a whole number by working in the context of length.

1 m – ☐ m = ☐ m

1 − 0·49 = ?</td>
<td>Use a place value grid to represent the stages of column subtraction, including exchanges where required.

5·74 − 2·25 = ?

 
</td>
<td>Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places.

3·921 − 3·75 = ?

O • Tth Hth Thth
3 • 9 2 1
− 3 • 7 5 0</td>
</tr>
<tr>
<td>Year 5 Multiplication</td>
<td colspan="3">Teaching point 1: For multiplication, if there is a multiplicative increase to one factor and a corresponding decrease to the other factor, the product stays the same

• Teaching point 2: Both the commutative law and the associative law can be applied when multiplying three or more numbers.

• Teaching point 3: The choice of which order to multiply in can be made according to the simplest calculation.
• Teaching point 4: Decimal fractions (with a whole number of tenths or hundredths) can be multiplied by a whole number by using known multiplication facts and unitising.

• Teaching point 5: Multiplying by 0.1 is equivalent to dividing by 10; multiplying by 0.01 is equivalent to dividing by 100. Understanding of place value can be used to divide a number by 10/100: when a number is divided by 10,</td>
</tr>
</table>

<table>
<tr><td></td><td colspan="3">the digits move one place to the right; when a number is divided by 100, the digits move two places to the right.

• Teaching point 6: To multiply a single-digit number by a decimal fraction with up to two decimal places, convert the decimal fraction to an integer by multiplying by 10 or 100, perform the resulting calculation using an appropriate strategy, then adjust the product by dividing by 10 or 100.

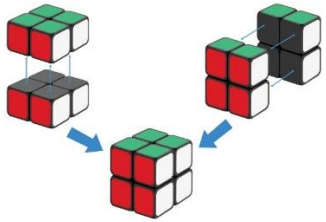
• Teaching point 7: If the multiplier is less than one, the product is less than the multiplicand; if the multiplier is greater than one, the product is greater than the multiplicand.

• Teaching point 8: Multiplication can be combined with addition and subtraction; when there are no brackets, multiplication is completed before addition or subtraction; when there are brackets, the calculation within the brackets is completed first.

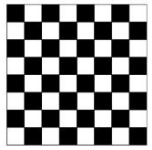
• Teaching point 9: When adding or subtracting multiplication expressions that have a common factor, the distributive law can be applied.</td></tr>
<tr><td>Understanding factors</td><td>Use cubes or counters to explore the meaning of ‘square numbers’.

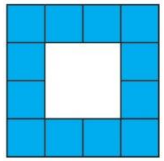
25 is a square number because it is made from 5 rows of 5.

Use cubes to explore cube numbers.

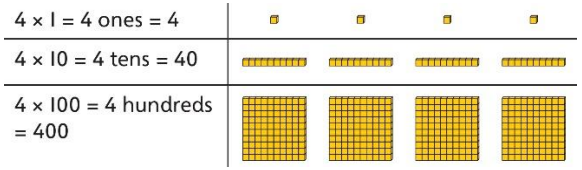
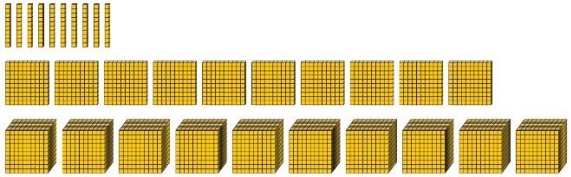
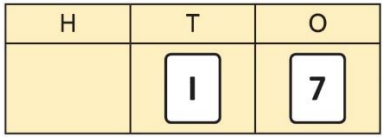
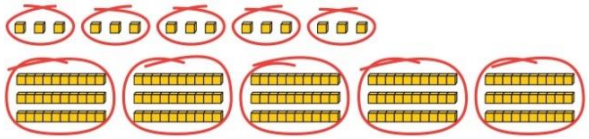
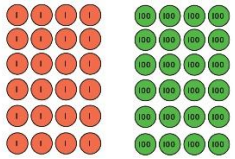
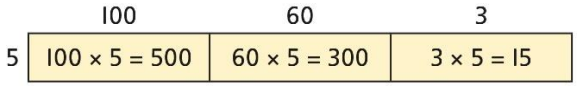
8 is a cube number.</td><td>Use images to explore examples and non-examples of square numbers.

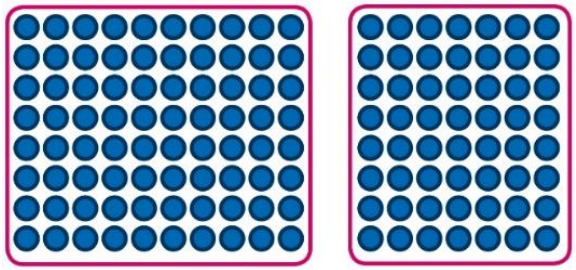
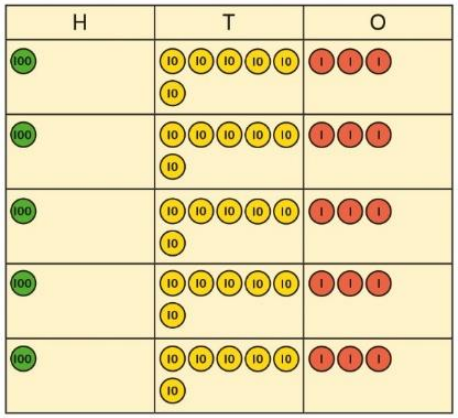
 
8 × 8 = 64
8^2 = 64

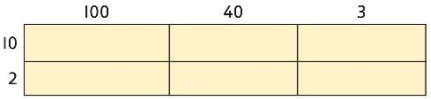
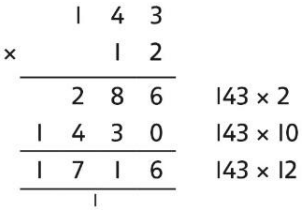
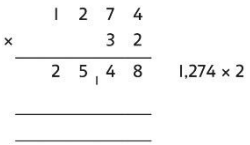
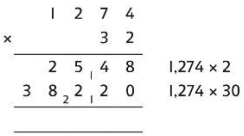
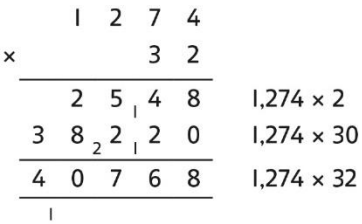
 

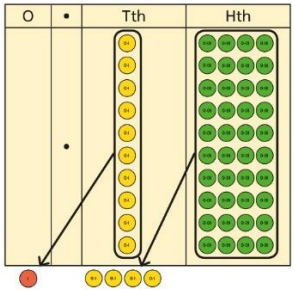
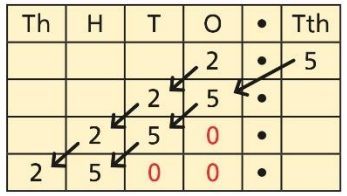
12 is not a square number, because you cannot multiply a whole number by itself to make 12.</td><td>Understand the pattern of square numbers in the multiplication tables.

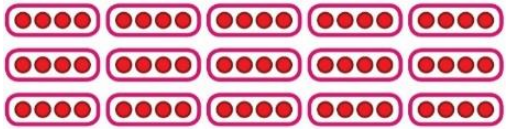
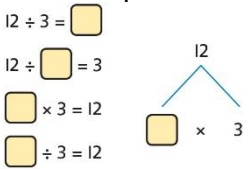
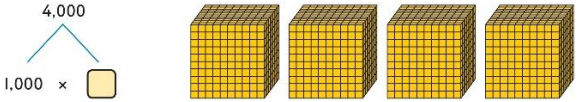
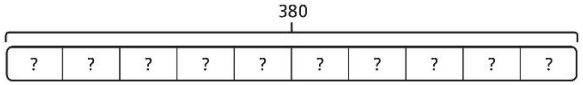
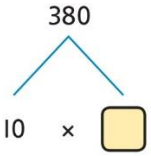
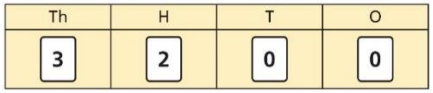
Use a multiplication grid to circle each square number. Can children spot a pattern?</td></tr>
</table>

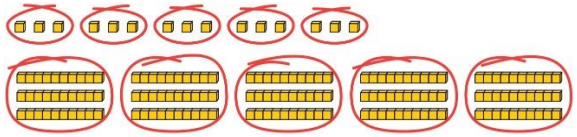
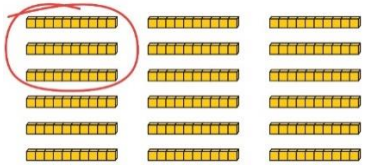
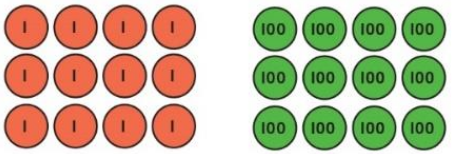
Multiplying by 10, 100 and 1,000	Use place value equipment to multiply by 10, 100 and 1,000 by unitising.  	Understand the effect of repeated multiplication by 10. 	Understand how exchange relates to the digits when multiplying by 10, 100 and 1,000.  *17 × 10 = 170* *17 × 100 = 17 × 10 × 10 = 1,700* *17 × 1,000 = 17 × 10 × 10 × 10 = 17,000*
Multiplying by multiples of 10, 100 and 1,000	Use place value equipment to explore multiplying by unitising.  *5 groups of 3 ones is 15 ones.* *5 groups of 3 tens is 15 tens.* *So, I know that 5 groups of 3 thousands would be 15 thousands.*	Use place value equipment to represent how to multiply by multiples of 10, 100 and 1,000.  *4 × 3 = 12* *4 × 300 = 1,200*  *6 × 4 = 24* *6 × 400 = 2,400*	Use known facts and unitising to multiply. *5 × 4 = 20* *5 × 40 = 200* *5 × 400 = 2,000* *5 × 4,000 − 20,000* *5,000 × 4 = 20,000*
Multiplying up to 4-digit numbers by a single digit	Explore how to use partitioning to multiply efficiently. *8 × 17 = ?*	Represent multiplications using place value equipment and add the 1s, then 10s, then 100s, then 1,000s.	Use an area model and then add the parts.  Use a column multiplication, including any required exchanges.

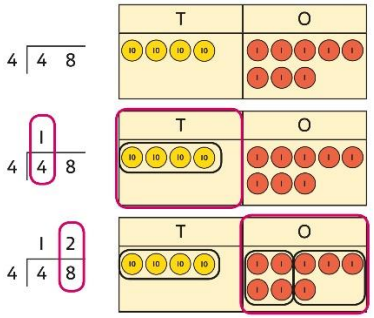
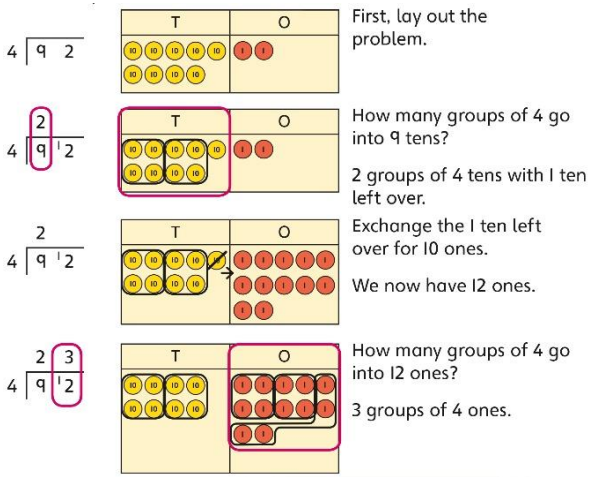
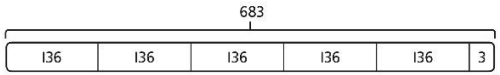
	 8 × 10 = 80 8 × 7 = 56 80 + 56 = 136 *So, 8 × 17 = 136*	 	1 3 6 × 6 8 1 6 2 3
Multiplying 2-digit numbers by 2-digit numbers	Partition one number into 10s and 1s, then add the parts. *23 × 15 = ?*  10 × 15 = 150 10 × 15 = 150  3 × 15 = 45 There are 345 bottles of milk in total. H T O 1 5 0 1 5 0 + 4 5 3 4 5 1 *23 × 15 = 345*	Use an area model and add the parts. *28 × 15 = ?* 20 m 8 m 10 m: 20 × 10 = 200 m² | 8 × 10 = 80 m² 5 m: 20 × 5 = 100 m² | 8 × 5 = 40 m² H T O 2 0 0 1 0 0 8 0 + 4 0 4 2 0 1 *28 × 15 = 420*	Use column multiplication, ensuring understanding of place value at each stage. 3 4 × 2 7 2 3 8 34 × 7 2 3 4 × 2 7 2 3 8 34 × 7 2 6 8 0 34 × 20 3 4 × 2 7 2 3 8 34 × 7 2 6 8 0 34 × 20 9 1 8 34 × 27 1
Multiplying up to 4-digits by 2-digits		Use the area model then add the parts.	Use column multiplication, ensuring understanding of place value at each stage.

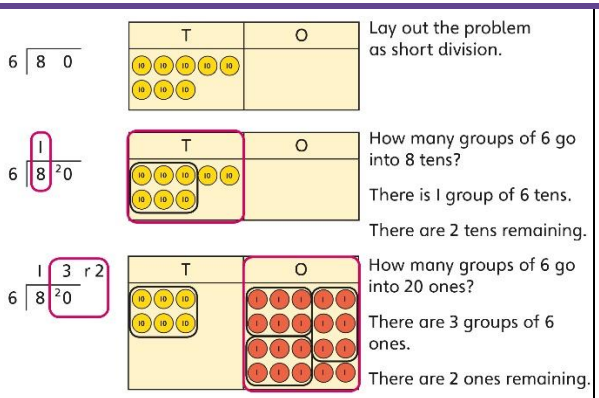
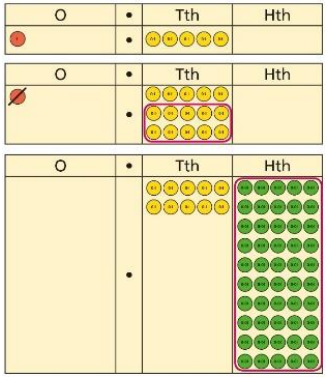
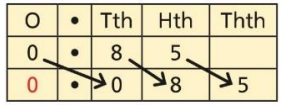
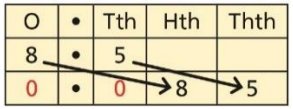
		 143 × 12 = 1,716 There are 1,716 boxes of cereal in total.  *143 × 12 = 1,716*	 Progress to include examples that require multiple exchanges as understanding, confidence and fluency build. *1,274 × 32 = ?* *First multiply 1,274 by 2.*  *Then multiply 1,274 by 30.*  *Finally, find the total.*  *1,274 × 32 = 40,768*
Multiplying decimals by 10, 100 and 1,000	Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.	Represent multiplication by 10 as exchange on a place value grid.	Understand how this exchange is represented on a place value chart.

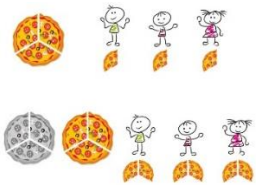
		 *0·14 × 10 = 1·4*	2·5 × 10 = 25 2·5 × 100 = 250 2·5 × 1,000 = 2,500 
Year 5 Division	• **Teaching point 1**: For division, if there is a multiplicative change to the dividend and a corresponding change to the divisor, the quotient stays the same. • **Teaching point 2**: To divide any decimal fraction with up to two decimal places by a single-digit number, convert the decimal fraction to an integer by multiplying by 10 or 100, perform the resulting calculation using an appropriate strategy, then adjust the quotient by dividing by 10 or 100.		
Understanding factors and prime numbers	Use equipment to explore the factors of a given number.  *24 ÷ 3 = 8* *24 ÷ 8 = 3* *8 and 3 are factors of 24 because they divide 24 exactly.* 24 ÷ 5 = 4 remainder 4.  *5 is not a factor of 24 because there is a remainder.*	Understand that prime numbers are numbers with exactly two factors. *13 ÷ 1 = 13* *13 ÷ 2 = 6 r 1* *13 ÷ 4 = 4 r 1* *1 and 13 are the only factors of 13.* *13 is a prime number.*	Understand how to recognise prime and composite numbers. *I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.* *I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.* *I know that 1 is not a prime number, as it has only 1 factor.*

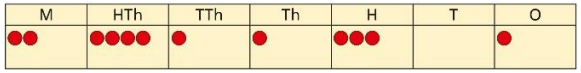
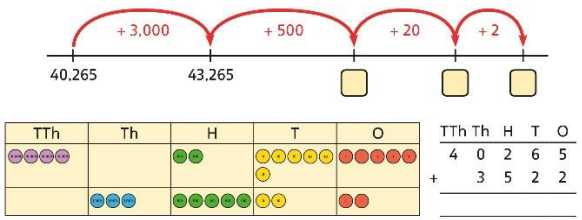
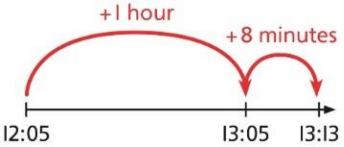
Understanding inverse operations and the link with multiplication, grouping and sharing	Use equipment to group and share and to explore the calculations that are present. *I have 28 counters.* *I made 7 groups of 4. There are 28 in total.* *I have 28 in total. I shared them equally into 7 groups. There are 4 in each group.* *I have 28 in total. I made groups of 4. There are 7 equal groups.*	Represent multiplicative relationships and explore the families of division facts.  *60 ÷ 4 = 15* *60 ÷ 15 = 4*	Represent the different multiplicative relationships to solve problems requiring inverse operations.  Understand missing number problems for division calculations and know how to solve them using inverse operations. *22 ÷ ? = 2* *22 ÷ 2 = ?* *? ÷ 2 = 22* *? ÷ 22 = 2*
Dividing whole numbers by 10, 100 and 1,000	Use place value equipment to support unitising for division. *4,000 ÷ 1,000*  *4,000 is 4 thousands.* *4 × 1,000= 4,000* *So, 4,000 ÷ 1,000 = 4*	Use a bar model to support dividing by unitising. *380 ÷ 10 = 38*   *380 is 38 tens.* *38 × 10 = 380* *10 × 38 = 380* *So, 380 ÷ 10 = 38*	Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.  *3,200 ÷ 100 = ?* *3,200 is 3 thousands and 2 hundreds.* *200 ÷ 100 = 2* *3,000 ÷ 100 = 30* *3,200 ÷ 100 = 32* *So, the digits will move two places to the right.*
Dividing by multiples of 10, 100 and 1,000	Use place value equipment to represent known facts and unitising.	Represent related facts with place value equipment when dividing by unitising.	Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check. *3,000 ÷ 5 = 600* *3,000 ÷ 50 = 60*

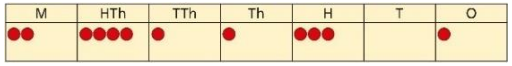
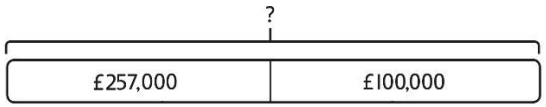
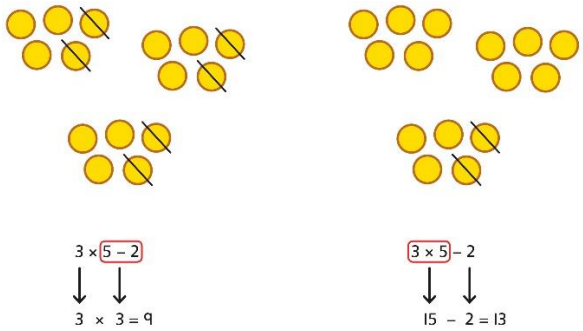
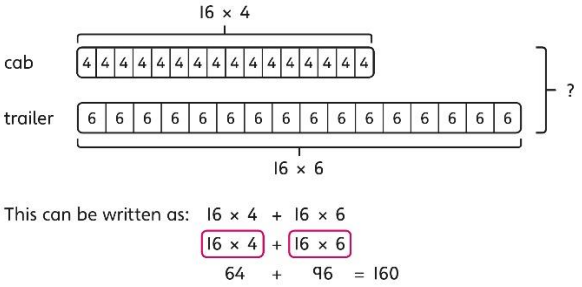
	 *15 ones put into groups of 3 ones. There are 5 groups.* *15 ÷ 3 = 5* *15 tens put into groups of 3 tens. There are 5 groups.* *150 ÷ 30 = 5*	 *180 is 18 tens.* *18 tens divided into groups of 3 tens. There are 6 groups.* *180 ÷ 30 = 6*  *12 ones divided into groups of 4. There are 3 groups.* *12 hundreds divided into groups of 4 hundreds. There are 3 groups.* *1200 ÷ 400 = 3*	*3,000 ÷ 500 = 6* *5 × 600 = 3,000* *50 × 60 = 3,000* *500 × 6 = 3,000*
Dividing up to four digits by a single digit using short division	Explore grouping using place value equipment. *268 ÷ 2 = ?* *There is 1 group of 2 hundreds.* *There are 3 groups of 2 tens.* *There are 4 groups of 2 ones.* *264 ÷ 2 = 134*	Use place value equipment on a place value grid alongside short division. The model uses grouping. A sharing model can also be used, although the model would need adapting.	Use short division for up to 4-digit numbers divided by a single digit. $\begin{array}{r} 0\ 5\ 5\ 6 \\ 7\overline{)\ 3\ {}^{3}8\ {}^{3}9\ {}^{4}2} \end{array}$ *3,892 ÷ 7 = 556* Use multiplication to check. *556 × 7 = ?*

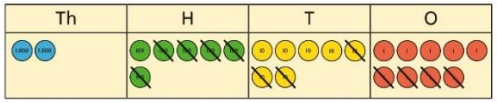
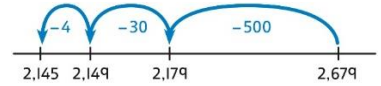
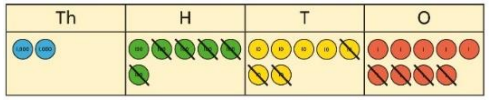
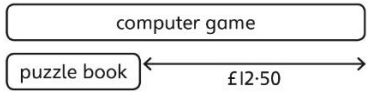
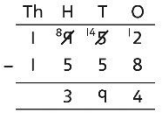
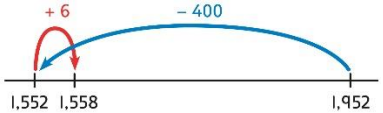
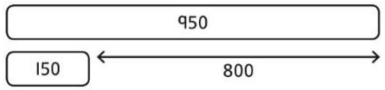
		 Lay out the problem as a short division. *There is 1 group of 4 in 4 tens.* *There are 2 groups of 4 in 8 ones.* Work with divisions that require exchange. 	*6 × 7 = 42* *50 × 7 = 350* *500 × 7 = 3500* *3,500 + 350 + 42 = 3,892*
Understanding remainders	Understand remainders using concrete versions of a problem. *80 cakes divided into trays of 6.* 	Use short division and understand remainders as the last remaining 1s.	In problem solving contexts, represent divisions including remainders with a bar model. 

	80 cakes in total. They make 13 groups of 6, with 2 remaining.		*683 = 136 × 5 + 3* *683 ÷ 5 = 136 r 3*
Dividing decimals by 10, 100 and 1,000	Understand division by 10 using exchange. *2 ones are 20 tenths.* *20 tenths divided by 10 is 2 tenths.*	Represent division using exchange on a place value grid.  *1·5 is 1 one and 5 tenths.* *This is equivalent to 10 tenths and 50 hundredths.* *10 tenths divided by 10 is 1 tenth.* *50 hundredths divided by 10 is 5 hundredths.* *1·5 divided by 10 is 1 tenth and 5 hundredths.*	Understand the movement of digits on a place value grid.  *0·85 ÷ 10 = 0·085*  *8·5 ÷ 100 = 0·085*

		1·5 ÷ 10 = 0.15	
Understanding the relationship between fractions and division	Use sharing to explore the link between fractions and division. *1 whole shared between 3 people.* *Each person receives one-third.* 	Use a bar model and other fraction representations to show the link between fractions and division.  $1 \div 3 = \frac{1}{3}$	Use the link between division and fractions to calculate divisions. $5 \div 4 = \frac{5}{4} = 1\frac{1}{4}$ $11 \div 4 = \frac{11}{4} = 2\frac{3}{4}$
Year 6			
	Concrete	**Pictorial**	**Abstract**
Year 6 Addition	• **Teaching point 1**: The digits in a number indicate its structure so it can be composed and decomposed. • **Teaching point 2**: Knowledge of crossing thousands boundaries can be used to work to and across millions boundaries. • **Teaching point 3**: Sometimes numbers are rounded as approximations to eliminate an unnecessary level of detail; rounded numbers are also used to give an estimate or average. At other times, precise readings are useful. • **Teaching point 4**: Fluent calculation requires the flexibility to move between mental and written methods according to the specific numbers in a calculation. • **Teaching point 5**: Problems with two unknowns can have one solution or more than one solution (or no solution). A relationship between the two unknowns can be described in different ways, including additively and multiplicatively. • **Teaching point 6**: Model drawing can be used to expose the structure of problems with two unknowns. • **Teaching point 7**: A problem with two unknowns has only one solution if the sum of the two unknowns and the difference between them is given (*'sum-and-difference problems'*) or if the sum of the two unknowns and a multiplicative relationship between them is given (*'sum-and-multiple problems'*).		

	• **Teaching point 8**: Other problems with two unknowns have only one solution. • **Teaching point 9**: Some problems with two unknowns can't easily be solved using model drawing but can be solved by a 'trial-and-improvement' approach; these problems may have one solution, several solutions or an infinite number of solutions.		
Comparing and selecting efficient methods	Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods. 	Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation. Compare written and mental methods alongside place value representations.  Use bar model and number line representations to model addition in problem-solving and measure contexts. 	Use column addition where mental methods are not efficient. Recognise common errors with column addition. *32,145 + 4,302 = ?* TTh Th H T O 3 2 1 4 5 + 4 3 0 2 3 6 4 4 7 TTh Th H T O 3 2 1 4 5 + 4 3 0 2 7 5 1 6 5 *Which method has been completed accurately?* *What mistake has been made?* Column methods are also used for decimal additions where mental methods are not efficient. H T O · Tth Hth 1 4 0 · 0 9 + 4 9 · 8 9 1 8 9 · 9 8 1
Selecting mental methods for	Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.	Use a bar model to support thinking in addition problems.	Use place value and unitising to support mental calculations with larger numbers.

larger numbers where appropriate	 *2,411,301 + 500,000 = ?* *This would be 5 more counters in the HTh place.* *So, the total is 2,911,301.* *2,411,301 + 500,000 = 2,911,301*	*257,000 + 99,000 = ?*  *I added 100 thousands then subtracted 1 thousand.* *257 thousands + 100 thousands = 357 thousands* *257,000 + 100,000 = 357,000* *357,000 – 1,000 = 356,000* *So, 257,000 + 99,000 = 356,000*	*195,000 + 6,000 = ?* *195 + 5 + 1 = 201* *195 thousands + 6 thousands = 201 thousands* *So, 195,000 + 6,000 = 201,000*
Understanding order of operations in calculations	Use equipment to model different interpretations of a calculation with more than one operation. Explore different results. *3 × 5 − 2 = ?* 	Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations. 	Understand the correct order of operations in calculations without brackets. Understand how brackets affect the order of operations in a calculation. *4 + 6 × 16* *4 + 96 = 100* *(4 + 6) × 16* *10 × 16 = 160*
Year 6 Subtraction			
Comparing and selecting efficient methods	Use counters on a place value grid to represent subtractions of larger numbers.	Compare subtraction methods alongside place value representations.	Compare and select methods. Use column subtraction when mental methods are not efficient.

		  Th H T O 2 6 7 9 − 5 3 4 2 1 4 5 Use a bar model to represent calculations, including 'find the difference' with two bars as comparison. 	Use two different methods for one calculation as a checking strategy.   Use column subtraction for decimal problems, including in the context of measure. H T O · Tth Hth 3 0 9 · 6 0 − 2 0 6 · 4 0 1 0 3 · 2 0
Subtracting mentally with larger numbers		Use a bar model to show how unitising can support mental calculations. *950,000 − 150,000* *That is 950 thousands − 150 thousands*  *So, the difference is 800 thousands.* *950,000 − 150,000 = 800,000*	Subtract efficiently from powers of 10. *10,000 − 500 = ?*
Year 6 Multiplication	• **Teaching point 1**: When multiplying two numbers that are multiples of 10, 100 or 1,000, multiply the number of tens, hundreds or thousands and then adjust the product using place value. • **Teaching point 2**: When multiplying two numbers where one number is a multiple of 10, 100 or 1,000, use short multiplication and adjust the product using place value.		

<table>
<tr><td></td><td colspan="3">

- **Teaching point 3**: Two two-digit numbers can be multiplied by partitioning one of the factors, calculating partial products and then adding these partial products. This method can be extended to multiplication of three-digit numbers by two-digit numbers.

- **Teaching point 4**: *'Long multiplication'* is an algorithm involving multiplication, then addition of partial products, which supports multiplication of two numbers with two or more digits.

- **Teaching point 5**: Multiplication where one of the factors is a composite number can be carried out by multiplying one factor and then the other factor.
- **Teaching point 6**: Any two- or three-digit dividend can be divided by a two-digit divisor by skip counting in multiples of the divisor (quotient < 10); these calculations can be recorded using the short or long division algorithms.

- **Teaching point 7**: Any three- or four-digit dividend can be divided by a two-digit divisor using the short or long division algorithms (including quotient ≥ 10).

- **Teaching point 8**: When there is a remainder, the result can be expressed as a whole-number quotient and a whole-number remainder, as a whole-number quotient and a proper-fraction remainder, or as a decimal-fraction quotient.
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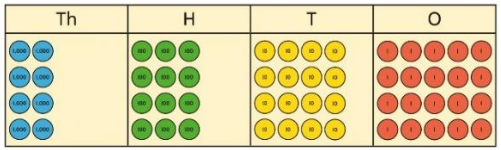
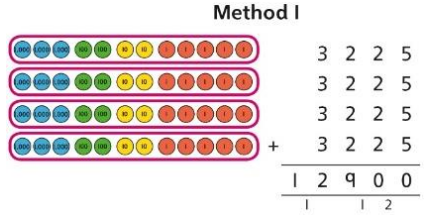
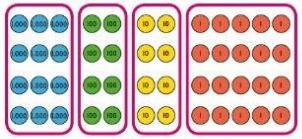
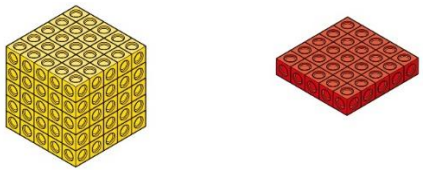
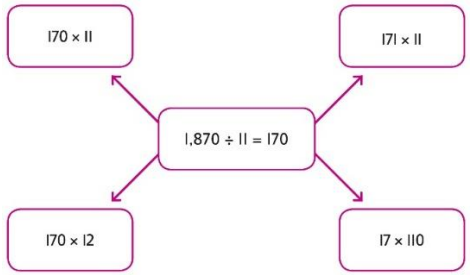
<tr><td>Multiplying up to a 4-digit number by a single digit number</td><td>Use equipment to explore multiplications.
 

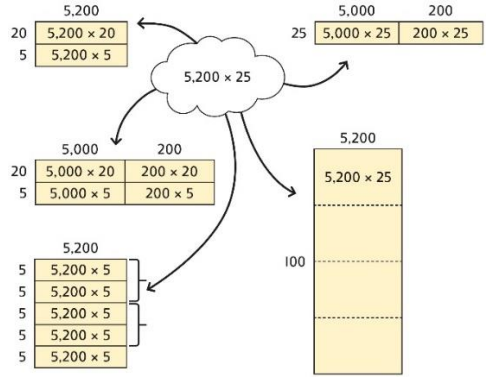
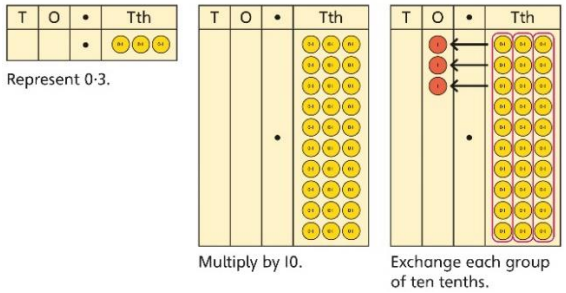
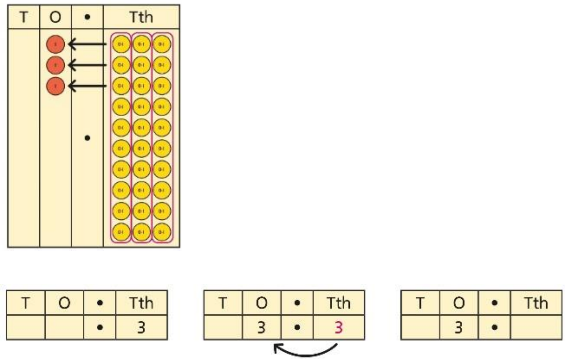
4 groups of 2,345

This is a multiplication:

4 × 2,345
2,345 × 4</td><td>Use place value equipment to compare methods.
 
</td><td>Understand area model and short multiplication.

Compare and select appropriate methods for specific multiplications.</td></tr>
</table>

		Method 2  4 × 3,000 4 × 200 4 × 20 4 × 5 12,000 + 800 + 80 + 20 = 12,900	**Method 3** 3,000 200 20 5 4 | 12,000 | 800 | 80 | 20 12,000 + 800 + 80 + 20 = 12,900 **Method 4** 3 2 2 5 × 4 1 2 9 0 0 1 2
Multiplying up to a 4-digit number by a 2-digit number		Use an area model alongside written multiplication. **Method 1** 1,000 200 30 5 20 | 20,000 | 4,000 | 600 | 100 1 | 1,000 | 200 | 30 | 5 1 2 3 5 × 2 1 5 1 × 5 3 0 1 × 30 2 0 0 1 × 200 1 0 0 0 1 × 1,000 1 0 0 20 × 5 6 0 0 20 × 30 4 0 0 0 20 × 200 2 0 0 0 0 20 × 1,000 2 5 9 3 5 21 × 1,235	Use compact column multiplication with understanding of place value at all stages. 1 2 3 5 × 2 1 1 2 3 5 1 × 1,235 2 4 7 0 0 20 × 1,235 2 5 9 3 5 21 × 1,235
Using knowledge of factors and partitions to compare methods for multiplications	Use equipment to understand square numbers and cube numbers.  $5 \times 5 = 5^2 = 25$ $5 \times 5 \times 5 = 5^3 = 25 \times 5 = 125$	Compare methods visually using an area model. Understand that multiple approaches will produce the same answer if completed accurately.	Use a known fact to generate families of related facts. 

		 Represent and compare methods using a bar model.	Use factors to calculate efficiently. *15 × 16* *= 3 × 5 × 2 × 8* *= 3 × 8 × 2 × 5* *= 24 × 10* *= 240*
Multiplying by 10, 100 and 1,000	Use place value equipment to explore exchange in decimal multiplication.  *0·3 × 10 = ?* *0·3 is 3 tenths.* *10 × 3 tenths are 30 tenths.* *30 tenths are equivalent to 3 ones.*	Understand how the exchange affects decimal numbers on a place value grid.  *0·3 × 10 = 3*	Use knowledge of multiplying by 10, 100 and 1,000 to multiply by multiples of 10, 100 and 1,000. *8 × 100 = 800* *8 × 300 = 800 × 3* *= 2,400* *2·5 × 10 = 25* *2·5 × 20 = 2·5 × 10 × 2* *= 50*
Multiplying decimals	Explore decimal multiplications using place value equipment and in the context of measures.	Represent calculations on a place value grid.	Use known facts to multiply decimals. *4 × 3 = 12* *4 × 0·3 = 1·2* *4 × 0·03 = 0·12* *20 × 5 = 100* *20 × 0·5 = 10*

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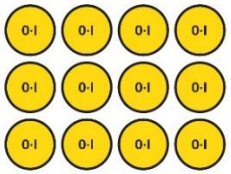
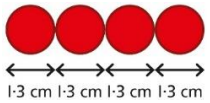


3 groups of 4 tenths is 12 tenths.
4 groups of 3 tenths is 12 tenths.



4 × 1 cm = 4 cm
4 × 0·3 cm = 1.2 cm
4 × 1·3 = 4 + 1·2 = 5·2 cm
</td>
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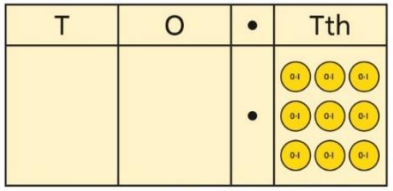
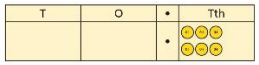
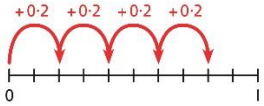
3 × 3 = 9

3 × 0·3 = 0·9



Understand the link between multiplying decimals and repeated addition.





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20 × 0·05 = 1

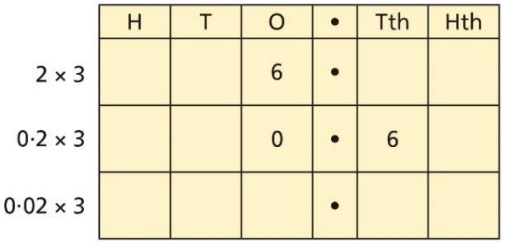
Find families of facts from a known multiplication.

I know that 18 × 4 = 72.

This can help me work out:

1·8 × 4 = ?
18 × 0·4 = ?
180 × 0·4 = ?
18 × 0·04 = ?

Use a place value grid to understand the effects of multiplying decimals.



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<td>Year 6
Division</td>
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Teaching point 1: Any two- or three-digit dividend can be divided by a two-digit divisor by skip counting in multiples of the divisor (quotient < 10); these calculations can be recorded using the short or long division algorithms.
Teaching point 2: Any three- or four-digit dividend can be divided by a two-digit divisor using the short or long division algorithms (including quotient ≥ 10).
Teaching point 3: When there is a remainder, the result can be expressed as a whole-number quotient and a whole-number remainder, as a whole-number quotient and a proper-fraction remainder, or as a decimal-fraction quotient.

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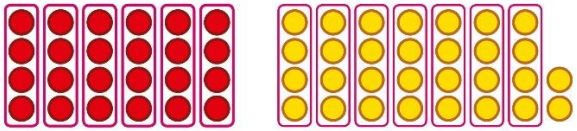
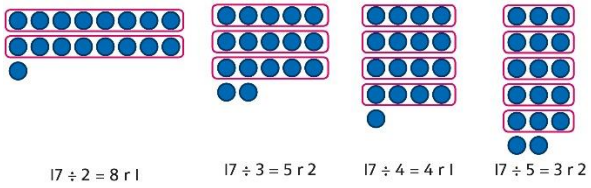
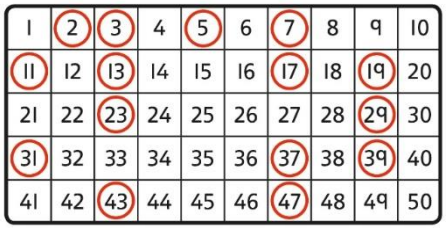
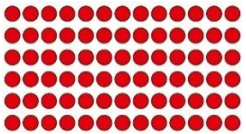
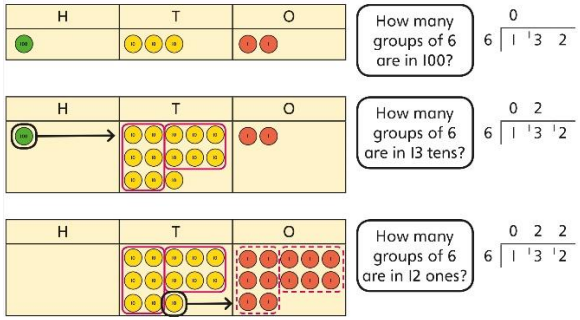
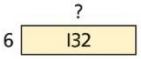
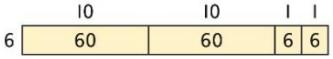
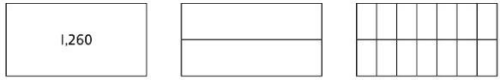
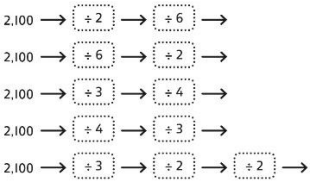
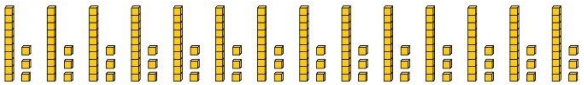
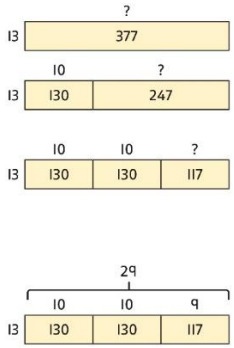
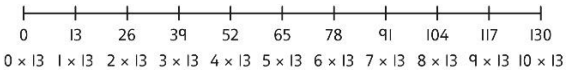
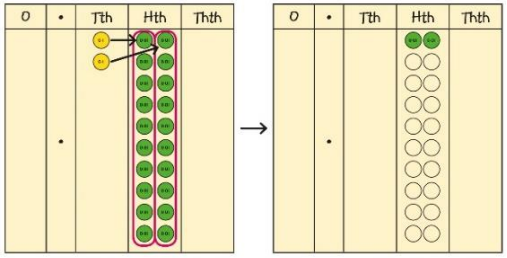
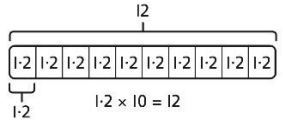
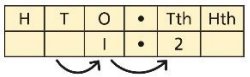
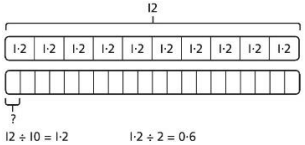
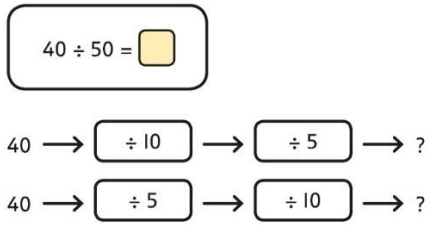
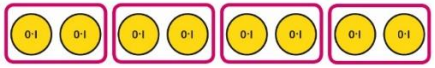
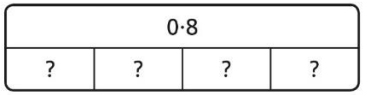
	• **Teaching point 4**: For division, if there is a multiplicative change to the dividend and the divisor remains the same, the quotient changes by the same scale factor. • **Teaching point 5**: For division, if there is a multiplicative increase to the divisor and the dividend remains the same, the quotient decreases by the same scale factor; if there is a multiplicative decrease to the divisor and the dividend remains the same, the quotient increases by the same scale factor. • **Teaching point 6**: Division can be combined with addition and subtraction; when there are no brackets, division is completed before addition or subtraction; when there are brackets, the calculation within the brackets is completed first. • **Teaching point 7**: When adding or subtracting division expressions that have a common divisor, the distributive law can be applied. •		
Understanding factors	Use equipment to explore different factors of a number.  *4 is a factor of 24 but is not a factor of 30.*	Recognise prime numbers as numbers having exactly two factors. Understand the link with division and remainders.  	Recognise and know primes up to 100. Understand that 2 is the only even prime, and that 1 is not a prime number. 
Dividing by a single digit	Use equipment to make groups from a total.  *There are 78 in total.* *There are 6 groups of 13.* *There are 13 groups of 6.*	 	Use short division to divide by a single digit.

			0 6 | 1 ¹3 2 0 2 6 | 1 ¹3 ¹2 0 2 2 6 | 1 ¹3 ¹2 Use an area model to link multiplication and division.  6 × ? = 132  132 = 120 + 12 132 ÷ 6 = 20 + 2 = 22
Dividing by a 2-digit number using factors	Understand that division by factors can be used when dividing by a number that is not prime.	Use factors and repeated division. *1,260 ÷ 14 = ?*  *1,260 ÷ 2 = 630* *630 ÷ 7 = 90* *1,260 ÷ 14 = 90*	Use factors and repeated division where appropriate. *2,100 ÷ 12 = ?* 
Dividing by a 2-digit number using long division	Use equipment to build numbers from groups.  *182 divided into groups of 13.* *There are 14 groups.*	Use an area model alongside written division to model the process. *377 ÷ 13 = ?*	Use long division where factors are not useful (for example, when dividing by a 2-digit prime number). Write the required multiples to support the division process. *377 ÷ 13 = ?*

		 *377 ÷ 13 = 29*	 13 ⟌ 3 7 7 − 1 3 0 10 2 4 7 − 1 3 0 10 1 1 7 − 1 1 7 9 0 29 *377 ÷ 13 = 29* A slightly different layout may be used, with the division completed above rather than at the side. 3 21 ⟌ 7 9 8 − 6 3 0 1 6 8 3 8 21 ⟌ 7 9 8 − 6 3 0 1 6 8 − 1 6 8 0 Divisions with a remainder explored in problem-solving contexts.
Dividing by 10, 100 and 1,000	Use place value equipment to explore division as exchange.	Represent division to show the relationship with multiplication. Understand the effect of dividing by 10, 100 and 1,000 on the digits on a place value grid.	Use knowledge of factors to divide by multiples of 10, 100 and 1,000.

	 *0·2 is 2 tenths.* *2 tenths is equivalent to 20 hundredths.* *20 hundredths divided by 10 is 2 hundredths.*	  Understand how to divide using division by 10, 100 and 1,000. *12 ÷ 20 = ?* 	 *40 ÷ 5 = 8* *8 ÷ 10 = 0·8* *So, 40 ÷ 50 = 0·8*
Dividing decimals	Use place value equipment to explore division of decimals.  *8 tenths divided into 4 groups. 2 tenths in each group.*	Use a bar model to represent divisions.  4 × 2 = 8 8 ÷ 4 = 2 So, 4 × 0·2 = 0·8 0·8 ÷ 4 = 0·2	Use short division to divide decimals with up to 2 decimal places.